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which are not periodical and cannot be used in the formulæ of reduction):

$$\delta A,_{\alpha} = \tan\delta A^{1}{}_{\alpha}\Delta_{1}\delta - \frac{2}{\sin 2\delta} A'{}_{\delta}\Delta_{1}\alpha$$

$$\delta A'{}_{\delta} = \tfrac{1}{2}\sin 2\delta A^{1}{}_{\alpha}\Delta_{1}\alpha.$$

If the systematic velocity be of the same order as the velocity of the Earth, these terms will be of the same magnitude as the corresponding terms in the annual aberration.

Uccle: May 5.

On a Parallel-Plate Double-Image Micrometer. By J. H. Poynting.

(*Communicated by Sir R. S. Ball.*)

If a ray of light passes through a plate of glass with parallel faces it emerges parallel to its original direction, but with any other than normal incidence it is shifted through a distance proportional nearly to the tangent of the angle of incidence, up to quite considerable values of that angle.

I have already published an account of a use of this property in obtaining a fine adjustment for a cathetometer telescope. (*Phil. Trans.* vol. 182 (1891), A., p. 588.) A parallel plate movable round a horizontal axis is placed just in front of the object-glass. The telescope is brought nearly to the level of the point to be sighted, and the final adjustment is completed by tilting the plate until the point appears on the cross wire.

We have for some time used a similar device with a microscope on the Mason College Physical Laboratory for the measurement of small objects, placing the parallel plate between the stage and objective. It is very easy and very rapid in use. The plate might be placed, for the measurement of smaller objects, between the objective and eyepiece, but we have never required the sensitiveness which would thus be obtained.

I have lately applied the principle in the construction of a double-image micrometer for a telescope, and as the instrument, as far as I can test it, appears to be successful, it may be worthy of description.

A circular parallel plate of glass, rather more than 1 inch in diameter and about $\frac{1}{8}$ inch thick, being cut down the middle, one half is fixed in one semicircle of a ring, while the other is placed in the other semicircle, attached, however, to an axis passing through a bearing in the ring, so that the plate can be rotated, the axis being in the plane of the fixed plate and perpendicular to its edge at the middle point. The ring is

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fixed in the tube of a 4-inch telescope of about 67 inches focal length, 16 inches in front of the focal plane, the division between the two plates being vertical. The axis of the movable plate passes out horizontally through the side of the tube, and a pointer is attached at right angles to it moving over a fixed straight scale, the pointer being nearly at the middle of the scale when the two plates are in one plane.

On looking at a star with the plates thus in one plane one image only is seen. But on moving the pointer the one plate is tilted and all the rays through it are shifted, so that two images appear, their separation being proportional nearly to the movement of the pointer along the scale. In the fixed position of the micrometer with the division between the plates vertical, it is possible only to measure vertical distances. For general use, of course, it should be possible to revolve it about the axis of the telescope. It was fixed merely for simplicity of construction in preparing it to test the method.

The separation of the images for a given angle of tilt might be calculated by the method given at the end of this note, knowing the exact thickness of the plate and its refractive index. But it appeared preferable to obtain the value per division of the scale by experiment. For this purpose the eyepiece of the telescope was removed and a fine scale of 500 divisions to the inch was fixed in the focal plane of the objective. Another telescope focussed for an infinite distance was then placed in front of the objective and the image of the fine scale was viewed by it. The movable plate was then tilted so that the lines of the fine scale in one image moved one place on, two places on, and so on, the positions of coincidence being easily determined. The tilts corresponded, of course, to displacements of the image, in the use of the micrometer, by successive 1-500ths of an inch. Within more than ten divisions on each side of the middle of the pointer scale the movements were proportional to the shifting of the fine scale, and the mean of a number of readings made twenty-one divisions of the pointer scale correspond exactly to eight divisions of the fine scale. The effective focal length of the objective might be taken as 67·16 inches, this being the distance from the near side of the flint glass to the focal plane. For exact work it would be necessary to calculate the position of the principal point for the system of lenses, but this would be quite needless refinement here. Hence

$$1 \text{ division of pointer scale} = \frac{8}{21} \times \frac{1}{500} \times \frac{1}{67{\cdot}16} \text{ radius, or } 2{\cdot}34 \text{ seconds.}$$

As ten divisions of the pointer scale occupy a length of 12·68 mm., and the scale is 67·5 mm. from the axis, ten divisions means an angle of tilt less than 11°, and up to this the error in taking the displacement as proportional to the movement of the pointer along the scale is not more than 1 in 600. I have found that with my telescope it is quite sufficient to move the pointer

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by hand, though with a steady stand and good definition a screw motion would be useful.

I have tested the micrometer by measuring the distance between the components of *Castor* when one is vertically over the other, and by measuring a vertical diameter of *Saturn*, using a power of 100. I have no experience of micrometer work, and do not know the accuracy to be expected, but, considering the very inferior definition of my telescope, and the unsteadiness of the stand, the results obtained appear to be hopeful for the success of the method.

The following are the results, in each case, of course, giving double the actual angle.

Distance between two components of Castor.

1892 *April* 22. *Readings of Pointer Scale.*

1st position with one component in one image on the other component in the other image.	2nd position with one component in one image on the other component in the other image.
34·2	28·6
33·1	28·1
33·1	28·1
33·1	27·7
32·9	28·5

Rejecting the first readings as made without experience, the mean difference is 4·95 divisions.

Hence angular separation of the components

$$\tfrac{1}{2} \times 4·95 \times 2·34 = 5''·79.$$

1892 *May* 9. *Readings of Pointer Scale.*

1st Position.	2nd Position.	1st Position.	2nd Position.
27·4	22·0	27·3	22·1
27·0	22·0	27·3	22·6
26·9	22·0	27·7	22·0
27·0	22·0	27·0	22·5
27·4	22·5	27·0	22·4

The mean difference is 4·99 divisions.

Hence angular separation of the components is

$$\tfrac{1}{2} \times 4·99 \times 2·34 = 5''·84.$$

In Denning's *Telescopic Work* (p. 304) the separation is given as 5″·68.

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VERTICAL DIAMETER OF SATURN.

1892 *April* 22. *Readings of Pointer Scale.*

1st position of movable image touching fixed image.	2nd position of movable image touching fixed image.
23·0	37·7
23·0	37·6
23·0	37·8

The mean difference is 14·7 divisions.

Hence vertical diameter is

$$\tfrac{1}{2} \times 14{\cdot}7 \times 2{\cdot}34 = 17''{\cdot}2.$$

I did not observe the angle the measured diameter made with the polar diameter, but from the westerly position of the planet I estimate it at over 30°. The polar diameter on this evening was

$$17''{\cdot}2,$$

so that the above is rather too small.

1892 *May* 9. *Readings of Pointer Scale.*

1st Position.	2nd Position.	1st Position.	2nd Position.
17·5	31·7	17·6	31·9
17·6	31·6	17·7	31·6
17·7	31·7		

The mean difference is 14·08 divisions.

Hence vertical diameter is

$$\tfrac{1}{2} \times 14{\cdot}08 \times 2{\cdot}34 = 16''{\cdot}5.$$

The diameter measured appeared to make about 20° with the polar diameter, which on this evening was

$$16''{\cdot}8.$$

Note on the calculated Value of the shifting of a Ray incident on a Parallel Plate.

Let the angle of incidence be ϕ, that of refraction ψ. Let t be the thickness of the plate, and μ its index of refraction. It is easily shown that the shift is

$$s = \frac{t \sin(\phi - \psi)}{\cos\psi}$$

$$= t \tan\phi \frac{\cos\phi \sin(\phi - \psi)}{\sin\phi \cos\psi}$$

$$= t \tan\phi \frac{(1 - \sin^2\phi)^{\frac{1}{2}}}{\sin\phi} \left\{ \sin\phi - \frac{(1 - \sin^2\phi)^{\frac{1}{2}} \sin\phi}{\mu \left(1 - \frac{\sin^2\phi}{u}\right)^{\frac{1}{2}}} \right\}$$

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$$\begin{aligned}
&= t\tan\phi\left\{(1-\sin^2\phi)^{\frac{1}{2}} - \frac{1-\sin^2\phi}{\mu}\left(1-\frac{\sin^2\phi}{\mu^2}\right)^{\frac{1}{2}}\right\}\\
&= t\tan\phi\left\{1-\frac{\sin^2\phi}{2}-\frac{\sin^4\phi}{8}-\frac{\sin^6\phi}{16}\ \ldots\right.\\
&\qquad\left.-\frac{1-\sin^2\phi}{\mu}\left(1+\tfrac{1}{2}\frac{\sin^2\phi}{\mu^2}+\tfrac{3}{8}\frac{\sin^4\phi}{\mu^4}+\tfrac{5}{16}\frac{\sin^6\phi}{\mu^6}+\right)\right\}\\
&= t\tan\phi\left\{\frac{\mu-1}{\mu}-\sin^2\phi\left(\frac{1}{2}-\frac{1}{\mu}+\frac{1}{2\mu^3}\right)\right.\\
&\qquad\qquad -\sin^4\phi\left(\frac{1}{8}-\frac{1}{2\mu^3}+\frac{3}{8\mu^5}\right)\\
&\qquad\qquad -\sin^6\phi\left(\frac{1}{16}-\frac{3}{8\mu^5}+\frac{5}{16\mu^7}\right)\\
&\qquad\qquad \left.-\ \&\text{c.}\right\}
\end{aligned}$$

Taking $\mu = \frac{3}{2}$ and ϕ successively 5°, 10°, and 20°, then approximately

$$s = \frac{t}{3}\tan 5^\circ\,(1-\cdot 00042)$$

$$s = \frac{t}{3}\tan 10^\circ\,(1-\cdot 00156)$$

$$s = \frac{t}{3}\tan 20^\circ\,(1-\cdot 00529).$$

The Lunar Eclipse, 1892 *May* 11: *Observations made at Mr. Crossley's Observatory, Bermerside, Halifax*. By Edward Crossley, M.P., and Joseph Gledhill.

The weather here on the 11th was all that could be desired—a cloudless day and night. Owing, however, to the low position of the Moon (altitude about 18° when on the meridian) the naked-eye and other phenomena were not well seen before 10 o'clock. But as the Moon got higher the spectacle was very fine: the eclipsed portion was seen to be of a clear ruddy tinge, and the principal lunar seas, &c., were readily seen and identified with the help of an opera-glass. This beautiful colour disappeared about midnight, and gave place to a dull grey tinge.

While watching for the occultations of the stars given in the *Observatory* for May, an attempt was made to note the times when the general curve of the dark shadow reached some of the best-known craters.

The instrument used throughout the observations was the $9\frac{1}{3}$-inch Cooke refractor, power 66.